Solution of the Dynamic Equations

The linear and the angular velocities of the bodies just after the impact can be calculated from Eqs (1–6), using the values of I_N and I_F determined by the graphical method

The graphical method and the solution of the equations are suitable for programming on a digital computer, and, thereby, it is possible to examine rapidly a large number of possible impacts

References

[1] Timoshenko, S and Young, D H, *Advanced Dynamics* (McGraw-Hill Book Co, Inc New York 1948)

[2] Routh, E J *The Elementary Part of Dynamics of a System of Rigid Bodies* (The Macmillan Co, New York, 1905)

A Note on "Pressure Distribution for Hypersonic Boundary-Layer Flow"

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THE pressure distribution of Ref 3 is improved here to represent, more accurately, the two cases of strong and weak hypersonic boundary-layer interaction phenomena for the derived flat-plate case from the wedge Incidentally, this also happens to be the pressure distribution of Hayes and Probstein[2] for the inviscid case The suitability of this pressure distribution for the viscous case can also be seen here

Analysis

From shock relations, we have

$$\frac{P_2 - P_1}{P_1} = \frac{2\gamma}{\gamma + 1}(M_1^2 \sin^2\theta - 1) \tag{1}$$

$$(M_1^2 \sin^2\theta - 1) = \frac{\gamma + 1}{2} M_1^2 \frac{\sin\theta \sin\Delta}{\cos(\theta - \Delta)} \tag{2}$$

$$\sin\Delta = (1 - \epsilon)\sin\theta\cos(\theta - \Delta) \tag{3}$$

where

P_2 = pressure behind the shock wave
P_1 = freestream pressure
γ = specific heats ratio
M_1 = freestream Mach number
θ = shock angle
Δ = deflection angle
ϵ = density ratio = $(\gamma - 1)/(\gamma + 1) + 2/[(\gamma + 1) M_1^2 \times \sin^2\theta]$

To satisfy the asymptotic conditions at $x = +\infty$ (x being the surface coordinate), let

$$\left.\begin{aligned}\theta &= \theta_0 + \theta_1\\ \Delta &= \Delta_0 + \Delta_1\end{aligned}\right\} \tag{4}$$

where θ_0 and Δ_0 are shock and deflection angles for the inviscid case, and θ_1 and Δ_1 correspond to shock and deflection angles for the viscous case; and so, after approximating $\cos(\theta - \Delta) \approx 1$, we have from relations (1–4)

$$\frac{P_2^1}{P_1} - 1 = \frac{\gamma M_1^2 \sin^2(\Delta_0 + \Delta_1)}{1 - \left\{\frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M_1^2 \sin^2(\theta_0 + \theta_1)}\right\}} \tag{5}$$

where P_2^1 is the pressure on the boundary layer

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The pressure distribution in relation (5) for the inviscid case, as represented by Hayes and Probstein,[2] is

$$\frac{P_2}{P_1} = 1 + \frac{\gamma M_1^2 \sin^2\Delta}{(1 - \epsilon)} \tag{6}$$

Before discussing the pressure distribution (5) further, we derive here the relations between θ_1 and Δ_1 for the special flat-plate case, placed along the freestream direction, from relation (3)

For the viscous case [Eq (3)], we have

$$(\Delta_0 + \Delta_1) = (\theta_0 + \theta_1) \times \left[1 - \frac{\gamma - 1}{\gamma + 1} - \frac{2}{(\gamma + 1)M_1^2(\theta_0^2 + 2\theta_0\theta_1 + \theta_1^2)}\right] \tag{7}$$

where

$$\sin(\Delta_0 + \Delta_1) \approx (\Delta_0 + \Delta_1)$$

$$\sin(\theta_0 + \theta_1) \approx (\theta_0 + \theta_1)$$

Relation (7), for the special case of $\theta_0 \approx \sin^{-1}(1/M_1)$ and $\Delta_0 \approx 0$, gives

$$M_1\Delta_1 = (1 + M_1\theta_1) \times \left[\frac{2}{\gamma + 1} - \frac{2}{(\gamma + 1)(1 + 2M_1\theta_1 + M_1^2\theta_1^2)}\right]$$

$$= (1 + M_1\theta_1)\left\{\frac{2}{\gamma + 1}\left[1 - \frac{1}{(1 + 2M_1\theta_1 + M_1^2\theta_1^2)}\right]\right\} \tag{8}$$

For the strong interaction case of $M_1\theta_1 \gg 1$, we have from relation (8) that

$$\Delta_1 \approx [2/(\gamma + 1)]\theta_1 \tag{9}$$

For the weak interaction case of $M_1\theta_1 \ll 1$, we have from relation (8)

$$M_1\Delta_1 = (1 + M_1\theta_1)\left\{\frac{2}{\gamma + 1} \times [1 - (1 - 2M_1\theta_1 - M_1^2\theta_1^2 + \quad)]\right\}$$

$$= (1 + M_1\theta_1)\left\{\frac{2}{\gamma + 1} M_1\theta_1[2 + M_1\theta_1 - \quad]\right\}$$

Therefore,

$$\Delta_1 \approx \frac{4}{\gamma + 1}\theta_1 \tag{10}$$

Hence, from pressure distribution (5), we have for the plate case

$$\frac{P_2^1}{P_1} - 1 = \frac{\gamma M_1^2 \Delta_1^2}{[2/(\gamma + 1)]\{1 - [1/(1 + 2M_1\theta_1 + M_1^2\theta_1^2)]\}} \tag{11}$$

For the strong interaction case we have, from relations (9) and (11)

$$\frac{P_2'}{P_1} - 1 = \frac{\gamma k^2}{\frac{2}{\gamma + 1}\left(1 - \frac{1}{\{1 + (\gamma + 1)k + [(\gamma + 1)/2]^2k^2\}}\right)}$$

where $k = M_1\Delta_1$ and which, for $k \gg 1$, becomes

$$P_2^1/P_1 = 1 + [\gamma(\gamma + 1)/2]k^2 \tag{12}$$

For the weak interactions case, we have, from relations (8) and (11),

$$\frac{P_2^1}{P_1} - 1 = \frac{\gamma M_1^2 \Delta_1^2}{(M_1\Delta_1)/(1 + M_1\theta_1)} = \gamma M_1 \Delta_1 (1 + M_1\theta_1) \quad (13)$$

From relations (10) and (13), we have, for the weak interaction case,

$$\frac{P_2^1}{P_1} = 1 + \gamma k + \frac{\gamma(\gamma + 1)}{4} k^2 \quad (14)$$

It may be of interest to note that the simplification of relation (11) with relation (10) will give us a series as

$$\frac{P_2'}{P_1} = 1 + \gamma k + \frac{\gamma(\gamma + 1)}{8} k^2 - \quad (15)$$

But this series, in relation (15), contains alternatingly negative and positive terms after the third term As such, this procedure may not be good, though the approximation can easily be made up to the third term

Conclusions

From relations (12) and (14), it can be seen that the pressure distribution in relation (5) is having a very good approximation of the tangent-wedge pressure distribution Also, it may be of interest to note that relations similar to relations (9) and (10) can be derived from relation (2) as well From the present note and the note in Ref 3, it can be seen that the pressure distribution of Pai in Ref 1 is well modified to bring the results very near to the tangent-wedge approximation

References

[1] Pai, S , ' Hypersonic viscous flow over an insulated wedge at an angle of attack, Univ of Maryland Rept BN42, Air Research and Development Command, Office of Scientific Research Rept OSR-TN-54 321 (October 1954)

[2] Hayes, W D and Probstein, R F , *Hypersonic Flow Theory* (Academic Press Inc , New York, 1959), p 280

[3] Sastry, M S , "Pressure distribution for hypersonic boundary-layer flow," AIAA J 1, 2398–2399 (1963)

An Analysis of the Yawing Motion of a Rocket with a Varying Mass

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Nomenclature

m = mass of the rocket as a function of time, slugs
m_0 = mass of the rocket at ignition, slugs
m_b = mass of the rocket at burnout, slugs
c = constant related to the burning rate of the propellant, per sec
t = time, sec
V = horizontal velocity of the rocket, fps
S = cross sectional area of the rocket, ft^2
T = thrust lb
k = radius of gyration, ft
r = distance from center of mass to nozzle exit, ft
l = length of the rocket, ft
C_d = aerodynamic drag coefficient
C_m = aerodynamic restoring moment coefficient
ρ = atmospheric density slugs/ft^3
φ = angle of yaw, rad

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THE angular oscillation about a horizontal flight path, experienced by a fin-stabilized rocket during the burning period, is known to have an increasing period and a decreasing amplitude This behavior is clearly due to the decreasing moment of inertia as the rocket loses mass and to the increasing magnitude of the restoring moment as the velocity of the rocket increases The authors have presented a simplified analysis which demonstrates the yaw behavior in a closed-form solution

The flight path of the rocket is horizontal, and the aerodynamic coefficients are assumed constant Although the mass is a function of time, it is assumed that the radius of gyration and the center of mass with respect to the vehicle remain unchanged The rate at which the propellant is consumed is constant, and the mass is represented as

$$m = m_0(1 - ct)$$

The equation of motion for the center of mass of the rocket is

$$m_0(1 - ct)(dV/dt) + \tfrac{1}{2}C_D\rho SV^2 = T$$

where the thrust is a constant This equation, of the Ricatti type, may be integrated by separation of variables to yield

$$V(t) = (T/\tfrac{1}{2}C_D\rho S)^{1/2}[C_0 + (1 - ct)^B]/[C_0 - (1 - ct)^B] \quad (1)$$

where

$$B = 2(\tfrac{1}{2}C_D\rho ST)^{1/2}/cm_0$$

$$C_0 = [V_0 + (T/\tfrac{1}{2}C_D\rho S)^{1/2}]/[V_0 - (T/\tfrac{1}{2}C_D\rho S)^{1/2}]$$

and V_0 is the horizontal velocity at the time of ignition

The equation describing the yaw is derived in Ref 1 and may be written as

$$m_0k^2(1 - ct)\ddot{\varphi} + (r^2 - k^2)cm_0\dot{\varphi} + \tfrac{1}{2}C_m\rho lSV^2\varphi = 0 \quad (2)$$

where the dot signifies the derivative with respect to time The angle of yaw is assumed small so that $\sin\varphi \approx \tan\varphi \approx \varphi$ Substituting for the velocity from Eq (1) and dividing by the leading coefficient gives

$$\ddot{\varphi} + \frac{c(r^2 - k^2)/k^2}{(1 - ct)}\dot{\varphi} + \frac{C_mlT/C_Dm_0k^2}{(1 - ct)}\left[\frac{C_0 + (1 - ct)^B}{C_0 - (1 - ct)^B}\right]^2 \varphi = 0 \quad (3)$$

For small rockets (final mass to initial mass ratios greater than 80%) the effect of the second term in Eq (3) is small in the final result; therefore, the coefficient is replaced by a constant average value $2K$ defined as

$$2K = \tfrac{1}{2}[c(r^2 - k^2)/k^2][(m_0 + m_b)/m_b]$$

The equation is then rewritten as

$$\ddot{\varphi} + 2K\dot{\varphi} + K_1^2[V^2(t)/(1 - ct)]\varphi = 0 \quad (4)$$

where

$$K_1^2 = \tfrac{1}{2}C_m\rho lS/m_0k^2$$

Furthermore, the variable coefficient of φ is approximated as

$$V^2(t)/(1 - ct) \simeq V_0^2(1 + t/T_b)^2$$

where

$$t_b/T_b = (1 + \Delta V/V_0)/(1 - \Delta m/m_0)^{1/2} - 1$$

and where t_b is the time of burnout, $\Delta V/V_0$ is the fractional velocity increase during the burning period, and $\Delta m/m_0$ is the fractional mass decrease over the same time Introducing the approximation into the equation yields

$$\ddot{\varphi} + 2K\dot{\varphi} + K_1^2V_0^2(1 + t/T_b)^2\varphi = 0$$